Radiatively-induced gravitational leptogenesis

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Abstract

We demonstrate how loop effects in gravitational backgrounds lead to a difference in the propagation of matter and antimatter, and show this is forbidden in flat space due to CPT and translation invariance. This mechanism, which is naturally present in beyond the standard model (BSM) theories exhibiting C and CP violation, generates a curvature-dependent chemical potential for leptons in the low-energy effective Lagrangian, allowing a matter-antimatter asymmetry to be generated in thermodynamic equilibrium, below the BSM scale.

1. Introduction

The origin of the matter-antimatter asymmetry of the universe remains one of the outstanding questions in particle physics and cosmology. Following the framework of the celebrated Sakharov [1] conditions, a popular and long-standing explanation has involved the out-of-equilibrium decay of heavy particles [2, 3], in which matter and antimatter are produced at different rates due to C and CP violation in the underlying theory. An alternative to this picture was proposed by Cohen and Kaplan [4] who noted that an asymmetry could, in fact, be generated *in equilibrium* through the coupling of C and CP violating operators involving the baryon or lepton currents to background fields, *e.g.*, $\partial_\mu \Phi j^\mu$ for a background scalar field. For isotropic background fields, this results in a chemical potential proportional to the time derivative $\dot{\Phi}$. More recently, Davoudiasl *et al.* [5] built on this idea by suggesting gravity could play the same role as Φ with an interaction $\partial_\mu R j^\mu$, where R is the Ricci scalar. Since then, many authors have gone on to postulate gravitational couplings as a means of generating matter asymmetry [6, 7, 8, 9, 10, 11, 12, 13]. However, with the exception of [6] (where the gravitational coupling arises from the axial anomaly), in almost all of these papers the required operators are introduced by hand, with no account of their dynamical origin, in the expectation that they may arise from some unspecified, more fundamental theory.

In this Letter, we present a new mechanism for gravitational leptogenesis in which the matter-antimatter asymmetry is generated dynamically at the quantum loop level, without the need to postulate additional interactions beyond the minimally coupled Lagrangian. Specifically, we show how in a C and CP violating theory, in which the light leptons are coupled to heavy states with mass M, the effective Lagrangian describing low-energy physics below this scale involves operators coupling directly to the background curvature, including the C and CP violating interaction $\partial_\mu R j^\mu / M^2$, which leads to a lepton-antilepton asymmetry.[1] [2] The coupling of C and CP violating operators to a time-dependent gravitational field circumvents the third Sakharov condition and allows the lepton-antilepton asymmetry to be generated in equilibrium.

The presence of explicit curvature-dependent operators in the effective Lagrangian represents a violation of the strong equivalence principle [14, 15]. The physical picture is that, at loop level, the light leptons propagate surrounded by a self-energy cloud of virtual parti-

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[1] A careful analysis of the modification to the dispersion relations implied by an operator $\partial_\mu R j^\mu / M^2$ has been given recently in [11], showing the same implications for lepton-antilepton asymmetry as follow from the interpretation of $\dot{R}$ as a chemical potential [4, 5].

[2] Another way to motivate the appearance of matter-antimatter asymmetry is to view $\partial_\mu R \sim \dot{R}$ as a fixed background coupling to the CPT odd current j^μ. In this sense, as originally presented in [4], the effect can be thought of as an "environmental CPT violation", with phenomenological consequences normally associated with a genuine breaking of CPT symmetry. The full operator $\partial_\mu R j^\mu$ is however CPT invariant. See [11] for a further discussion.

cles, including the heavy states. This virtual cloud has a length scale of order $1/M$ and so interacts with the background gravitational field through tidal, curvature-dependent forces, while its composition encodes the dynamics and symmetries of the heavy particles. In this way, gravity probes the physics of the high-scale fundamental theory and transmits this information to the low-energy effective Lagrangian describing the light leptons.

The existence of C and CP violating operators coupling to the curvature leads directly to a difference in the propagation of matter and antimatter. Of course, this would be inconsistent with the (strong) equivalence principle and, in particular, could not occur in flat space. In sec. 2, we give a formal proof that CPT and translation invariance forbids this situation for interacting theories in Minkowski space, regardless of whether there is any C (or CP) violation in the theory, which is of course a *necessary* condition for asymmetric propagation. Conversely, when gravity, C and CP violation are present, we show there is indeed a difference in the propagation of matter and antimatter. Only if all these conditions are met will this happen, meaning that the effect is intrinsic to gravitational backgrounds and not simply a consequence of C and CP violation already present in the original Lagrangian.

The mechanism described here is very general. For clarity, however, we illustrate it in a specific model familiar in the BSM literature, namely the "see-saw" Lagrangian, in which the light, left-handed lepton doublets ℓ_i ($i = e, \mu, \tau$) and Higgs field[3] are coupled to heavy right-handed sterile neutrinos N_α with non-degenerate masses M_α ($\alpha = 1, \dots n$):

$$\mathcal{L} = \sqrt{-g}\left[\overline{N}\not{D}N + \lambda_{i\alpha}\bar{\ell}_i\phi N_\alpha + \frac{1}{2}\overline{(N^c)}\, M\, N + \text{h.c.}\right] . \tag{1}$$

$\lambda_{i\alpha}$ is a complex Yukawa matrix, providing the required C and CP violation. For clarity, we omit any explicit labelling of L and R handed fields in what follows.

This is simply the model used by Fukugita and Yanagida [2] in their original demonstration of leptogenesis in flat space through the out-of-equilibrium decays of the heavy neutrinos, $N_\alpha \to \ell_i\phi^*$. Rather than using the heavy neutrinos in this way, however, we integrate them out to obtain the low-energy effective Lagrangian describing the physics of the leptons ℓ_i below the BSM scale M_α. In this Letter, we show that this gives rise to the operator

$$\mathcal{L}_i = \partial_\mu R\, \bar{\ell}_i\gamma^\mu\ell_i \sum_{\alpha,\beta,j} \frac{\text{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right]}{3M_\alpha M_\beta} I_{[\alpha\beta]} \tag{2}$$

The function $I_{\alpha\beta} = I\left(M_\alpha, M_\beta\right)$, which we calculate exactly, gives an antisymmetric part under interchange of M_α and M_β and is determined from a certain class of two-loop self-energy diagrams. Eq.(2) is of exactly the form required to generate a lepton-antilepton asymmetry. This is maintained by $\Delta L = 2$, $\phi\ell^c \leftrightarrow \phi^*\ell$ reactions in equilibrium which, in the conventional heavy-decay model, can wash out the lepton asymmetry but are essential in our scenario. We therefore have a mechanism for radiatively-induced gravitational leptogenesis, in which the asymmetry can be generated in equilibrium long after the decay of the heavy particles, at energies and temperatures well below their mass.

2. Propagation and CPT

In a C invariant theory, the propagation of matter and anti-matter will be identical, so the presence of complex λ is crucial to have an asymmetry in matter/anti-matter propagation, regardless of the background. We now show that the propagation of matter and antimatter must be the same in any theory in which translation and CPT symmetry holds. We demonstrate this explictly for spin 1/2 Dirac fermions. CPT symmetry is realised by an anti-unitary operator Θ such that the lepton propagator satisfies

$$\begin{aligned} S_{ab}(x', x) &= \langle \ell_a(x')\bar{\ell}_b(x)\rangle \\ &= \left\langle\left(\Theta\ell_a(x')\Theta^{-1}\right)\left(\Theta\bar{\ell}_b(x)\Theta^{-1}\right)\right\rangle^* , \end{aligned} \tag{3}$$

where a, b label spinor components. The CPT transformations can be written as $\Theta\ell(x')\Theta^{-1} = \gamma^0\gamma_5 C^{-1}\ell^c(-x)$ and $\Theta\bar{\ell}(x')\Theta^{-1} = \overline{\ell^c}(-x')C\gamma_5\gamma^0$, where $\ell^c = C\bar{\ell}^T$ is the Dirac charge conjugate and C is the charge-conjugation matrix satisfying $C\,(\gamma^\mu)^T\, C^{-1} = -\gamma^\mu$. Inserting these expressions, and taking note of the overall complex conjugation, we find, after some algebra

$$S(x', x) = \gamma_5 C[S^c(-x, -x')]^T C^{-1}\gamma_5 , \tag{4}$$

where $S^c(x, y) = \langle\ell^c(x)\overline{\ell^c}(y)\rangle$ is the antiparticle propagator. Translation symmetry means that $S^c(x, y) = S^c(x - y)$ which implies that $S^c(-x, -x') = S^c(x', x)$. From Lorentz invariance (inherent to a discusssion of

[3] In this notation, the Higgs doublet $\tilde{\phi}$ appearing in the SM lepton sector is related by $\phi^a = \epsilon^{ab}\tilde{\phi}^{\dagger b}$.

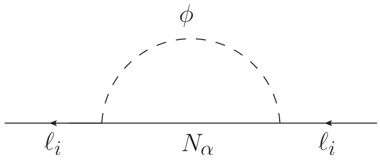

Figure 1: One-loop lepton self-energy

spinors) we can write

$$S^c(x',x) = S^c(x'-x) = \int \frac{d^d p}{(2\pi)^d}\left[A(p^2)\not{p} + B(p^2)\right] e^{-ip\cdot(x'-x)} \tag{5}$$

for some functions A and B. Substituting this expression into (4) and using the properties of the matrix C gives

$$S(x',x) = S^c(x',x)\,, \tag{6}$$

establishing that matter and antimatter propagate identically in a translational invariant and CPT conserving theory.

We now examine how loop corrections in gravitational backgrounds, which in general violate translation symmetry, can create a difference in lepton and antilepton self-energies $\Sigma(x,x') - \Sigma^c(x,x')$ associated to the propagators $\langle \ell(x)\overline{\ell}(x')\rangle$ and $\langle \ell^c(x)\overline{\ell^c}(x')\rangle$.

First, note that in the model of Eq.(1), the Majorana mass term for the heavy neutrinos means that there are two classes of propagators, *charge-violating* propagators $S^\times_\alpha(x,x') = \langle N(x)\overline{N^c}(x')\rangle$ and *charge-conserving* propagators $S_\alpha(x,x') = \langle N_\alpha(x)\overline{N}_\alpha(x')\rangle$ where the C script denotes the Dirac charge conjugate. In flat space, translation invariance allows us to write them in momentum space as

$$S_\alpha(p) = \frac{i\not{p}}{p^2 - M_\alpha^2}, \qquad S^\times_\alpha(p) = \frac{iM_\alpha}{p^2 - M_\alpha^2}\,. \tag{7}$$

As we see below, the charge violating propagators are key to generating a matter-antimatter asymmetry.

At one loop (see figure 1), the lepton and anti-lepton propagators are the same:

$$\Sigma_i(x,x') = \Sigma^c_i(x,x') = \sum_\alpha \lambda^\dagger_{\alpha i}\lambda_{i\alpha} G(x,y) S_\alpha(x,y)\,. \tag{8}$$

However, at two loops there are two diagrams (figure 2), which give non-zero contributions to $\Sigma(x,x') - \Sigma^c(x,x')$. For instance, in the case of the charge violating heavy

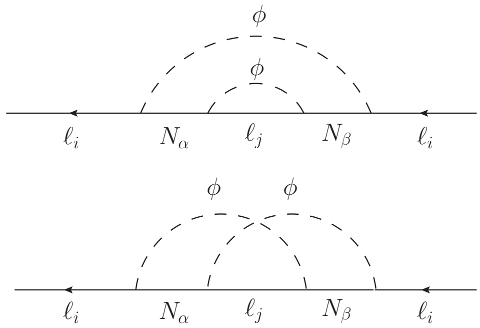

Figure 2: Two-loop corrections to lepton self-energies giving non-zero contributions to $\Sigma - \Sigma^c$.

neutrino propagators, the first diagram gives

$$\begin{aligned}\Sigma_i(x,x') - \Sigma^c_i(x,x') &= \sum_{\alpha,\beta,j} \mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right] \\ &\times G(x,x') \int d^4y \int d^4z\; G(y,z) S^\times_{[\alpha}(x,y) S_j(y,z) S^\times_{\beta]}(z,x')\,,\end{aligned} \tag{9}$$

whilst the second gives

$$\begin{aligned}\Sigma_i(x,x') - \Sigma^c_i(x,x') &= \sum_{\alpha,\beta,j} \mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right] \\ &\times \int d^4y \int d^4z\, G(y,x') G(x,z) S^\times_{[\alpha}(x,y) S_j(y,z) S^\times_{\beta]}(z,x')\,.\end{aligned} \tag{10}$$

Notice that we have antisymmetrised over α and β in the integral since $\mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right]$ is antisymmetric in α,β. For the other type of heavy neutrino propagator, only the first diagram contributes (due to charge considerations) and the expression is similar to (9) but with a Yukawa matrix contribution $\mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\alpha j}\lambda_{j\beta}\right]$. It is now clear that Eqs.(9) and (10) are non-vanishing in curved spacetime. We therefore see that as a consequence of breaking translation invariance by a general background, there is a difference in the propagation of matter and antimatter at two loops.

Given the general proof above, we must also find that if we restore translation invariance by going to Minkowski space, (9) and (10) will vanish. Indeed, substituting the flat space propagators of (7), we see explicitly that the integral is symmetric under interchange of α and β, and $\Sigma - \Sigma^c = 0$ as expected.

3. An effective action for leptons

We study the dynamics of leptons at the quantum loop level using an effective action in curved spacetime, valid at energies below the heavy neutrino mass scale M_α, *i.e.* we integrate out the heavy neutrinos.

The fundamental physics of how gravity affects the propagation of particles in curved backgrounds at loop level is now well understood (see, *e.g.* [11, 14, 16, 17, 18]). As an interacting particle propagates, it becomes surrounded by a screening cloud of virtual particles, acquiring an effective size and, as a result, experiences tidal forces from background curvature. Hence, the effective action, which captures the effect of quantum loops, will involve interactions between particle fields and background curvature. The fundamental Lagrangian respects the strong equivalence principle, by virtue of minimal coupling to gravity through the connection only, so particles and antiparticles propagate identically at tree level. However, the interaction of the gravitational field with this virtual cloud violates strong equivalence, causing the dynamics to become sensitive to the background curvature at loop level. As a result, the effective lagrangian will contain strong equivalence violating operators which couple the curvature tensor to lepton fields, allowing – depending on the structure of the cloud – the generation of C and CP violating operators such as $\partial_\mu R\bar{\ell}\gamma^\mu\ell$.

Since we are interested in the propagation of leptons, we consider an effective action which is quadratic in the lepton field, so that tidal effects manifest themselves as couplings between the Riemann tensor $R_{\mu\nu\rho\sigma}$ (and its various contractions) and fermion bilinears $\bar{\ell}(\cdots)\ell$. The most general such action, consistent with the symmetries of the tree-level action, namely general covariance and gauge symmetry, was discussed in detail in [11]. To leading order in the mass dimension of the couplings, it consists of operators of the form

$$\begin{aligned}\mathcal{L}_{eff} = \sqrt{-g}\Big[&\bar{\ell}i\not{D}\ell + ia\bar{\ell}\Big(2R_{\mu\nu}\gamma^\mu D^\nu + \frac{1}{2}\partial_\mu R\gamma^\mu\Big)\ell \\ &+ b\partial_\mu R\bar{\ell}\gamma^\mu\ell \\ &+ ic\bar{\ell}\left(2R\not{D} + \partial_\mu R\gamma^\mu\right)\ell \\ &+ id\bar{\ell}\Big(2D^2\not{D} + \frac{1}{4}\partial_\mu R\gamma^\mu\Big)\ell\Big], \quad (11)\end{aligned}$$

where a, b, c, d are real effective couplings of mass dimension minus two, which will depend on $\lambda_{i\alpha}$ and the masses m_H and M_α in the loops. There is one term in this effective action which is of great importance for leptogenesis and is the only C and CP violating operator in (11), *viz.*

$$\mathcal{L}_{CPV} = b\,\partial_\mu R\,\bar{\ell}\gamma^\mu\ell\,. \quad (12)$$

A careful discussion of the action of C, P and T on each of the operators appearing in $\mathcal{L}_{eff}$ is given in [11].

We compute the effective coupling b by matching the full and effective theories. We can capitalise on the fact that the effective couplings are independent of the choice of background and work in a conformally flat metric

$$g_{\mu\nu} = \Omega^2\eta_{\mu\nu} = (1+h)\eta_{\mu\nu}\,, \quad (13)$$

which is sufficient to distinguish the various components of the effective Lagrangian (11). The computation is also simplified if we work with conformally rescaled fields,

$$N \to \Omega^{-(n-1)/2}N, \quad \ell \to \Omega^{-(n-1)/2}\ell, \quad \phi \to \Omega^{-(n-2)/2}\phi. \quad (14)$$

After conformal rescaling, gravity enters only via

$$\mathcal{L}_\Omega = \frac{1}{2}\Omega\overline{N^c}MN + \Omega^2\left(m_H^2 - \frac{R}{6}\right)\phi^\dagger\phi + \Omega^{-(n-4)/2}\lambda_{i\alpha}\bar{\ell}_i\phi N\,, \quad (15)$$

where $R = -3\partial^2\Omega^2$ is the Ricci scalar for (13). This can then be expanded to linear order in h to give $\mathcal{L}_\Omega = h(x)\mathcal{O}(x)$. The effective couplings can be computed by matching the transition matrix elements $\langle\,\ell(p')\,|\,\mathcal{O}\,|\,\ell(p)\,\rangle$ to the effective amplitudes (see in particular [16, 11], as well as [14, 19, 20], for more details). Since $R = -3\partial^2 h$, the contribution to the effective vertex from the operator $\mathcal{L}_{CPV} = b\partial_\mu R\bar{\ell}\gamma^\mu\ell$, gives a contribution of the form shown in figure 3.

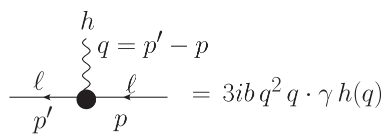

Figure 3: The effective h vertex, where $q = p' - p$ is the momentum transfer between the ingoing and outgoing lepton

For phenomenological reasons related to leptogenesis (which we will explain below), we are only interested in diagrams which involve the charge-violating neutrino propagators $\langle N(x)\overline{N^c}(x')\rangle$. For this kind of heavy neutrino propagator, there are in fact no additional contributions from h at the Yukawa vertex $\mathcal{L}_\lambda = -\frac{1}{2}(n-4)h\lambda_{i\alpha}\bar{\ell}_i\phi N_\alpha$. The reason is that this term only contributes for diagrams whose UV divergences produce a pole $1/(n-4)$ to cancel the $(n-4)$ pre-factor. Since $S_\alpha^\times(x,x') = M_\alpha/(p^2 - M_\alpha^2)$ is more strongly UV

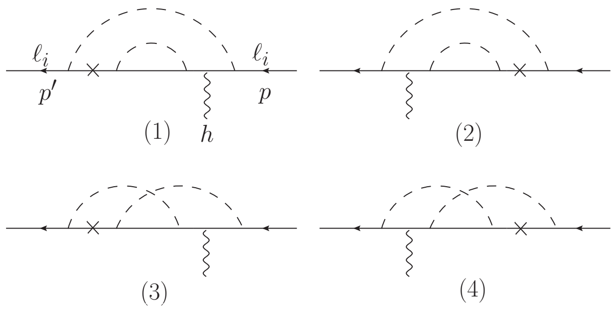

Figure 4: Contributions to $\langle \underline{\ell}(p') | O | \ell(p) \rangle$ from the heavy neutrino mass term $\frac{1}{2} h \bar{N} M N^c$. The cross in *e.g.*, (1) denotes the $S_\alpha^\times$ sterile neutrino propagator, and at the h vertex, there are contributions $S_\beta S_\beta$ and $S_\beta^\times S_\beta^\times$ corresponding to each propagator type.

convergent than $S_\alpha(p) = \not{p}/(p^2 - M_\alpha^2)$, the two loop diagrams involving the first kind of propagator contain very few UV divergences. In fact, the vertex correction diagram is UV finite, with degree of divergence $D = -1$, whilst the propagator correction diagram contains a single pole $1/(n-4)$, arising from the propagator correction sub-diagram, which is removed by subtracting an appropriate counterterm during renormalisation.

The only remaining terms in (15) which contribute are the heavy neutrino mass term, and the $\phi^\dagger \phi$ Higgs interactions. A full discussion of the these effective Lagrangian calculations will be presented elsewhere [21]. Here, we focus on the contributions to $\langle \ell(p') | O | \ell(p) \rangle$ from the heavy neutrino couplings to h shown in figure 4. The contribution from these diagrams to the $iq^2 \not{q}$ term is:

$$\langle \ell_i(p') | O | \ell_i(p) \rangle = iq^2 \not{q}\, h(q) \sum_{\alpha,\beta,j} \frac{\mathrm{Im}\left[\lambda^\dagger_{\beta i} \lambda_{i\alpha} \lambda^\dagger_{\beta j} \lambda_{j\alpha}\right]}{M_\alpha M_\beta} I_{[\alpha\beta]} \,, \tag{16}$$

where $I_{\alpha\beta} = I(M_\alpha, M_\beta)$, and i labels the lepton generation. Note that $I_{\alpha\beta}$ must have a non-vanishing antisymmetric part for (16) to be non-zero. The contributions to I from each diagram are rather involved and the complete set of results will be given in [21]. As an illustration, we quote here the result from diagram (1) to demonstrate explicitly the appearance of a non-vanishing contribution to $I_{[\alpha\beta]}$. We find

$$I^{(1)}_{[\alpha\beta]} = F(r) + G(r) \ln\left[\frac{\mu}{M_\alpha + M_\beta}\right] \tag{17}$$

where μ is the mass scale of dimensional regularisation, $r = (M_\alpha - M_\beta)/(M_\alpha + M_\beta)$ and

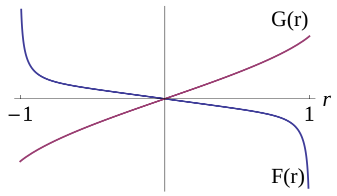

Figure 5: The antisymmetric functions contributing to $I_{[\alpha\beta]}$, with $-1 \le r \le 1$.

$$\begin{aligned} F(r) = {} & \frac{1}{384(4\pi)^4 r^4}\Bigg[12r\left(2r^2-1\right) - 3\left(r^2-1\right)^2 \ln^2\left(\frac{1-r}{1+r}\right) \\ & - 2\left(2r\left(5r^2-3\right) - 3\left(r^2-1\right)^2 \ln\left(\frac{1-r}{1+r}\right)\right)\ln\left(\frac{1-r}{2}\right) \\ & - 2\left(4r^4 - 5r^3 - 7r^2 + 3r + 3\right)\ln\left(\frac{1-r}{1+r}\right)\Bigg] \\ G(r) = {} & \frac{1}{192(4\pi)^4 r^4}\left[2r\left(5r^2-3\right) - 3\left(r^2-1\right)^2 \ln\left(\frac{1-r}{1+r}\right)\right] \end{aligned} \tag{18}$$

Antisymmetry under interchange of M_α and M_β is now manifest from the anti-symmetry of $F(r)$ and $G(r)$ under $r \to -r$ shown in figure 5.

We have, therefore, shown by explicit calculation that the operator $\partial_\mu R \bar{\ell}_i \gamma^\mu \ell_i$ is indeed generated, for each lepton flavour with the effective interaction being given by comparing (16) with figure 3:

$$\mathcal{L}_i = \partial_\mu R \left(\bar{\ell}_i \gamma^\mu \ell_i\right) \sum_{\alpha,\beta,j} \frac{\mathrm{Im}\left[\lambda^\dagger_{\beta i} \lambda_{i\alpha} \lambda^\dagger_{\beta j} \lambda_{j\alpha}\right]}{3 M_\alpha M_\beta} I_{[\alpha\beta]} \,. \tag{19}$$

This demonstrates that a combination of background curvature, complex couplings (*i.e.* C and CP violation) and loop effects can generate a leptogenesis-inducing operator. The dependence of (19) on the non-degeneracy of sterile neutrino masses is discussed in figure 6.

4. Consequences for leptogenesis

We now describe how this radiatively induced operator leads to a mechanism of leptogenesis and why the other class of diagrams, with charge-conserving heavy neutrino propagators, do not. In isotropic spacetimes, the interaction (19) has the form of a chemical potential

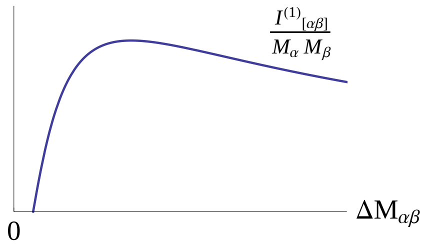

Figure 6: For fixed $M_\beta < M_\alpha$, C and CP violation from $\mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right]$ is initially enhanced as the mass difference $\Delta M_{\alpha\beta} = M_\alpha - M_\beta$ increases from zero. It then reaches a maximum before tending to zero, as radiative effects become mass-suppressed when $M_\alpha \to \infty$.

μ_i between matter and antimatter for each lepton generation given by

$$\mu_i = \dot{R} \sum_{\alpha,\beta,j} \frac{\mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right]}{3M_\alpha M_\beta} I_{[\alpha\beta]} \,. \qquad (20)$$

If T is the temperature of the early universe, this creates a lepton asymmetry of the form

$$n(\ell_i) - n(\ell_i^c) = \dot{R}\, T^2 \sum_{\alpha,\beta,j} \frac{\mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\beta j}\lambda_{j\alpha}\right]}{3M_\alpha M_\beta} I_{[\alpha\beta]} \,. \quad (21)$$

Summing over all lepton generations, the total lepton asymmetry ($L = \sum_i \ell_i$) is given by

$$n(L) - n(L^c) = \dot{R}\, T^2 \sum_{\alpha,\beta} \frac{\mathrm{Im}\left[\left(\lambda^\dagger\lambda\right)^2_{\alpha\beta}\right]}{3M_\alpha M_\beta} I_{[\alpha\beta]} \,. \qquad (22)$$

The formula (22) is the centrepiece of this Letter. It captures how three effects conspire to generate matter-antimatter asymmetry: the breaking of (time) translational symmetry by gravity in $\dot{R}$, C and CP violation from $\mathrm{Im}\left[\left(\lambda^\dagger\lambda\right)^2_{\alpha\beta}\right]$ and quantum loop effects in $I_{\alpha\beta}$. In particular, this mechanism remains active at energies and temperatures below the heavy scale and so is able to generate an asymmetry *after* the heavy neutrino decays, where the asymmetry is maintained in equilibrium by the $\Delta L = 2$ reactions $\phi\ell^c \leftrightarrow \phi^*\ell$.

Now that we have revealed the bigger picture, we are able to explain why the 2 loop contributions involving the charge-conserving propagators $\langle N(x)\overline{N}(x')\rangle$ are of less interest for leptogenesis. If we had instead calculated contributions from diagrams with this type of propagator, we would have found a different Yukawa matrix structure in the amplitude, leading to a generational lepton asymmetry

$$n(\ell_i) - n(\ell_i^c) = \dot{R}\, T^2 \sum_{\alpha,\beta,j} \mathrm{Im}\left[\lambda^\dagger_{\beta i}\lambda_{i\alpha}\lambda^\dagger_{\alpha j}\lambda_{j\beta}\right] J_{[\alpha\beta]} \,. \quad (23)$$

While this gives an asymmetry for each flavour, summing over all generations gives $n(L) - n(L^c) \propto \sum_{\alpha,\beta} \mathrm{Im}\left[(\lambda^\dagger\lambda)_{\beta\alpha}(\lambda^\dagger\lambda)_{\alpha\beta}\right] J_{[\alpha\beta]}$. However, $\mathrm{Im}\left[(\lambda^\dagger\lambda)_{\beta\alpha}(\lambda^\dagger\lambda)_{\alpha\beta}\right] = \mathrm{Im}\left|\left(\lambda^\dagger\lambda\right)_{\beta\alpha}\right|^2 = 0$, and so the total lepton asymmetry from these diagrams is zero.

5. Discussion

In this Letter, we have presented a new mechanism – radiatively-induced gravitational leptogenesis – for generating matter-antimatter asymmetry. We have shown how leptons and antileptons can propagate differently in curved spacetime due to gravitational interactions with their self-energy cloud of virtual high-mass particles. This effect is forbidden in flat space by CPT and translation invariance, and at tree-level in curved spacetime, by the strong equivalence principle. At loop level, however, the strong equivalence principle no longer holds and, depending on the composition of the cloud, C and CP violating operators can be generated in the low-energy effective Lagrangian. A simple interpretation in terms of a chemical potential for leptons shows immediately that this generates an asymmetry in the equilibrium distributions of matter and antimatter.

As already noted, this mechanism is very general, and its implementation in the specific Fukugita-Yanagida model described here is just one example. In particular, it arises naturally in most existing models of leptogenesis, which typically involve a high-energy BSM sector with C and CP violation, where it generates a matter-antimatter asymmetry at low energies and temperatures after the decay and decoupling of the heavy particles.

The next step is therefore to implement this mechanism within specific phenomenologies, *e.g.* GUT, SUSY and other leptogenesis models, giving a more thorough analysis of kinetic aspects of these theories. This would involve a discussion of Boltzmann equations, decoupling temperatures, reaction rates and the strength of curvature at various times in the Universe's history, *e.g.* inflation, radiation, matter domination. For instance, work is currently under way [22] to study the present leptogenesis model in warm inflation, where both temperature and curvature are high. Such analyses will allow us to see in what situations this mechanism can quantitatively account for the observed matter-antimatter asymmetry in the Universe.

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